

(4) Kittel 6.4

For non-relativistic particles, $E = p^2/2m$

For relativistic particles $E = \sqrt{(mc^2)^2 + (pc)^2} = pc\sqrt{1 + \left(\frac{mc^2}{pc}\right)^2}$

If $pc \gg mc^2$, $\left(\frac{mc^2}{pc}\right)^2 \ll 1$ and we can ignore it.

So $E = pc$ for our problem.

From (3.58), the wave-functions that are allowed are

$$\Psi(x, y, z) = A \sin(k_x x) \sin(k_y y) \sin(k_z z), \text{ where}$$

$$k_x = \frac{n_x \pi}{L}, n_x = 1, 2, \dots; \text{ same for } k_y \text{ and } k_z$$

(Essentially this result comes from the condition that the wavefunction, which determines the probability density of a particle to be found at a given location, vanishes outside the box in which the particle lives; so Ψ is a standing wave along each of the three dimensions)

From quantum mechanics, $p_x = \hbar k_x$, same for p_y and p_z

$$(In the book, this result is expressed as $p = \frac{\hbar}{\lambda}$; so, in particular, $p_x = \frac{\hbar}{\lambda_x} = \frac{\hbar}{(2\pi/k_x)} = k_x \cdot \frac{\hbar}{2\pi} = \hbar k_x)$$$

$$\text{Therefore, } p_x = \frac{\hbar n_x \pi}{L} = \frac{\hbar n_x \pi}{2\pi L} = \frac{\hbar n_x}{2L}, \text{ same for } p_y \text{ and } p_z$$

$$\text{Hence, } p = \sqrt{p_x^2 + p_y^2 + p_z^2} = \frac{\hbar}{2L} \sqrt{n_x^2 + n_y^2 + n_z^2}$$

$$\text{and } E = pc = \frac{\hbar c}{2L} \sqrt{n_x^2 + n_y^2 + n_z^2}$$